NUMBER CONNECTIONS puzzles by Rodolfo Kurchan and Claudio Meller

Connect pairs of numbers, covering the entire board and without passing through the same square twice.

All different numbers that appear in each board should have one connection.

For N x M boards which are the disposition with most quantity of solutions?

You can use the quantity of N different numbers you want, but at least 2 of each.

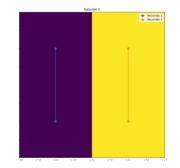
For example in 4 x M boards you should use at least 2 numbers 1, 2, 3 and 4.

Are this the best solutions?

OEIS 2 x M: 1, 5, 22, 54, 134,

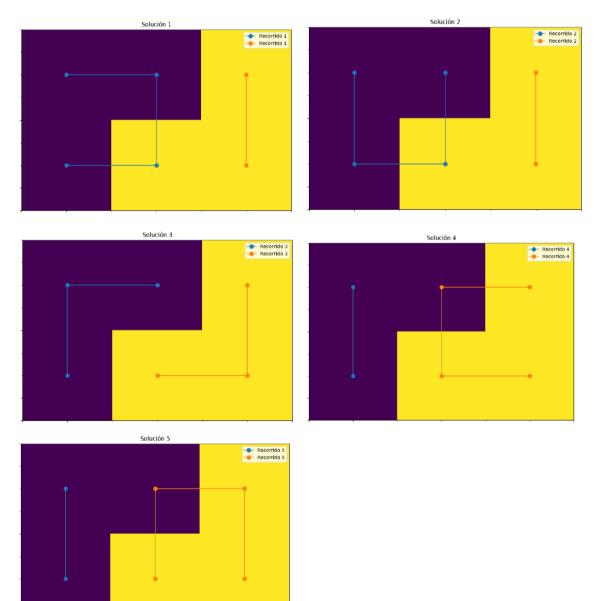
2x2 = 1

1	2
1	2



1	1	2
1	2	2

Example of the 5 solutions:



Another 2x3 = 5 solution:

2x3 = 5

1	2	2
1	2	2

2x4 = 22

1	1	2	2
1	1	2	2

2x5 = 54

1	1	1	2	2
1	1	2	2	2

2x5 = 54

1	1	2	2	2
1	1	2	2	2

2x6 = 134

1	1	1	2	2	2
1	1	1	2	2	2

Are this the best solutions?

OEIS 3 x M: 6, 72, 274, 134, 1910,

3x3 = 6

1	3	3
1	2	2
1	2	2

3x4 = 72

1	1	3	3
1	1	3	3
2	2	2	2

3x5 = 274

1	1	1	3	3
1	1	3	3	3
2	2	2	2	2

3x6 = 1910

1	1	2	2	3	3
1	1	2	2	3	3
1	1	2	2	3	3

OEIS 4 x M: 800,

4x4: 800

1	1	2	2
1	1	2	2
3	3	4	4
3	3	4	4

4x5:?

OEIS 5 x M: ?,

5x5:?

1	1	2	2	2
1	1	2	2	2
1	1	5	4	4
3	3	3	4	4
3	3	3	5	5