Problems & Puzzles: Puzzles

Puzzle 249. From Rudolf to Rodolfo (magic squares and pandigital numbers)

In 1989 **Rudolf Ondrejka** (JMR, 21, Vol.1) asked:

what is the magic square with the smallest magic sum using only pandigital numbers?

Rodolfo Marcelo Kurchan, from Buenos Aires, Argentina, found (year?) the following answer to the **Ondrejka**'s challenge:

1037956284	1036947285	1027856394	1026847395
1026857394	1027846395	1036957284	1037946285
1036847295	1037856294	1026947385	1027956384
1027946385	1026957384	1037846295	1036857294

Pandigital magic sum = 4129607358

Kurchan says that he found his solution without using computer.

I found this magic square at the page 237 of the **C. A. Pickover**'s '**Wonders of numbers**'. But you can see it also in one of the **Kurchan**'s pages at the web.

Pickover writes:

"*He* [**Kurchan**] *believes that this is the smallest nontrivial magic square having* n^2 *distinct pandigital* (*) *integers and having the smallest pandigital magic sum*".

I think that this is not so; probably the above shown magic square is the smallest magic 4x4 of that type, but it must exist some 3x3 solution.

As a matter of fact I have gotten without too much pain (because I used my PC and codes ;-) a 3x3 solution of the same type just disregarding the pandigital magic sum condition:

1023856974 1032857469 1028356479
1032856479 1028356974 1023857469
1028357469 1023856479 1032856974

Magic sum = 3085070922 (non pandigital)

I suspect that near to this one it should exist another solution with a pandigital magic sum (but I might be wrong!)

Question 1. Find the smallest 3x3 magic square as the Kurchan' s 4x4 one (if it exist!).

Question 2. Find a 3x3 magic square using only primes each having all the ten digits at least once and with the magic sum of the same type (but composite, of course!).

(*)**pandigital means here that all ten digits are used and 0 is not a leading digit.**

Solution:

For the Question 1 contributions came from **Rodolfo Marcelo Kurchan, C. Rivera, J. C. Rosa and Jon Wharf**.

Only **C. Rivera** and **J. C. Rosa** discovered technically at the same time and independently, the asked (minimal) solution to Question 1.

Nobody has sent specific solutions to Question 2.

A Happy and unexpected note! **Rodolfo Marcelo Kurchan** was contacted by email and sent an improved solution by himself obtained recently, for the 4x4 pandigital magic square with pandigital magic sum.

1034728695	1035628794	1024739685	1025639784
1024639785	1025739684	1034628795	1035728694
1035629784	1034729685	1025638794	1024738695
1025738694	1024638795	1035729684	1034629785

Pandigital magic sum = 4120736958. He says that German Gonzalez-Morris told him that this was now the smallest (just for the 4x4 case, as you will learn in short).

German Gonzalez-Morris added (May 2006) that he made a computer program and found an smaller pandigital sum (4120967358) then Rodolfo (by hand) found the smallest sum (4120736958), finally German found (and prove by exhaustive search) all smallest sums beginning from: 4120736958, 4120953678, 4120967358, 4127360958, 4129536078, ...

Here are their contributions in large.

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C. Rivera wrote:

As a matter of fact, as I suspected there is one smaller (than the Kurchan's one) pandigital magic sum solution in a magic 3x3 square:

1057834962 1084263579 1063549278
1074263589 1068549273 1062834957
1073549268 1052834967 1079263584
Pandigital Magical sum = 3205647819

I got it this Sunday morning (4/1/04). It was pretty close enough the one reported before when I posed this puzzle the Saturday morning. So, my PC just worked 24 hours more and bingo!. **By the method employed (exhaustive and upward) this must be the minimal solution.**

Other solutions after the minimal one and still less that the Kurchan one (shown in increasing pandigital magic sum) are:

1089362475 1320589746 1204968537
1320579648 1204973586 1089367524
1204978635 1089357426 1320584697
Pandigital Magical sum = 3614920758

1084793625 1327405896 1205349687
1326405798 1205849736 1085293674
1206349785 1084293576 1326905847
Pandigital Magical sum = 3617549208

1085793462 1328405679 1206349578
1327405689 1206849573 1086293457
1207349568 1085293467 1327905684
Pandigital Magical sum = 3620548719

1045793862 1368405279 1206349578
1367405289 1206849573 1046293857
1207349568 1045293867 1367905284
Pandigital Magical sum = 3620548719

1045798362 1368420579 1206359478
1367420589 1206859473 1046298357
1207359468 1045298367 1367920584
Pandigital Magical sum = 3620578419

And my PC is still working on...

Notes:

a) Please observe my 4th and 5th solution: they share the same pandigital magical sum!

b) But the problem posed by **Ondrejka** is a kind of old (15 years!), so I also suspect that someone else should have gotten the minimal solution before and of course that I'll be glad to publish the name of the first discoverer properly referenced...

***

J. C. Rosa wrote:

Today (Wednesday 7/1/04) is a magic day. I have found the smallest 3x3 magic square with the smallest magic sum using only pandigital numbers . Here it is :

1079263584 1052834967 1073549268
1062834957 1068549273 1074263589
1063549278 1084263579 1057834962
Magic sum =3205647819

Now , I'm looking for the largest....

***

Jon Wharf wrote:

After thinking about active groups of digits in a magic square and playing with bits of paper for ages, I generated the 5820 10-digit pandigital numbers which are also 10-digit pandigital when multiplied by 3.

So pretty quickly after that I found one solution:

1720945863 1270946853 2170946358

2170946853 1720946358 1270945863

1270946358 2170945863 1720946853

with pandigital magic constant 5162839074.

Minimum? no, but at least we're started....

Next solution uncovered was:

1283604759 1238704659 1328654709

1328704659 1283654709 1238604759

1238654709 1328604759 1283704659

with pandigital magic constant 3850964127. This was the smallest I found. It has the definite virtue of a smaller magic constant than **Rodolfo**'s.

***

J. C. Rosa wrote (March 23, 2005):

I have found (at last !) a solution to the question 2,

but I think that this solution maybe is not the smallest ...

10887852687493	10245252478639	10575552896347
10257252896347	10569552687493	10881852478639
10563552478639	10893852896347	10251252687493

magic sum=31708658062479

***

Later, on May 5, 2005 he wrote too:

About the question 2 of the puzzle 249 I have found several solutions smaller than the one already published. Here is my best solution (with 9 prime pandigital numbers of 12 digits each):

914052876349	106438267459	510267485239
106467485239	510252876349	914038267459
510238267459	914067485239	106452876349

magic sum=1530758629047

I think that this solution is not the smallest but now...I stop the search ...

Puzzle 252. Kurchan squares

Let's remember first what a magic square is.

"a magic square is a square array of integer numbers such that the **sum** of the numbers of each row, column and (main) diagonal **is a constant**"

For example, for a square 3x3 filled with the first 9 natural numbers (1 to 9), there is only one magic square

8	**1**	**6**
3	**5**	**7**
4	**9**	**2**

constant = 15

The key word for a magic square is the **sum** operation. But what are the **multiplication values** of the elements for the same magic square, for each row, column and (main) diagonal, equal or unequal? The answer is that **in general the multiplication values are not equal to a constant**.

			120
8	**1**	**6**	48
3	**5**	**7**	105
4	**9**	**2**	72
96	45	84	80

Now let's calculate the difference between the **maximal** and the **minimal** of these eight products and we will get 75 = 120 -45.

Let's define K(n) for a square array nxn as the difference of the maximal and the minimal products for each row, column and (main) diagonal.

Is there a square array 3x3 such that **K(3) is itself a minimal quantity**, let's say K°(3), when filled with the first n^2 natural numbers?

This exactly the question that was posed by **Rodolfo Kurchan** (1989) whose answer, given by himself, is K°(3)=72, and the corresponding 3x3 square is this one:

			126
8	**1**	**7**	56
4	**6**	**5**	120
3	**9**	**2**	54
96	54	70	96

K°(3)=126-54=72

I will call this kind of squares - **filled with the first n^2 X-type of numbers and having a minimal K°(n) value** - ***Kurchan multiplicative squares*** or shortly a ***Kurchan squares*** (*)

He solved also the same question filling the square with the first n^2 prime numbers

19 2 13

5	**11**	**17**
7	**23**	**3**

K°(3) = 518

While **Kurchan** says in his email (10/1/04) that this last answer may be improved, I verified exhaustively his two answers and I can assure that he has gotten the minimal K°(3) solutions for both ways of filling the 3x3 square (natural numbers and prime numbers).

Here is the **Kurchan** question:

Q1. Find K°(n) for n=4-10 for both ways of filling the squares (the first n^2 natural numbers and the first n^2 prime numbers)

Now I want to add four (4) questions.

More interested in the method than in the results, and -of course- avoiding the exhaustive approaches...

Q2. ...do you devise a smart approach in order to get the K°(n) values and the corresponding squares?

I have obtained specific squares - filled with the first n^2 natural numbers - such that K(4)=188 and K(5)=3680. I'm almost sure that my K(4) is K°(4) and then it can not be improved, but perhaps my K(5) is not yet the proper K°(5), so probably it can be improved.

Q3. Can you improve my K°(n) values for n=4 and 5, and/or get the specific squares associated?

Two more and last issues related to the **Kurchan** squares are the following ones:

Q4. Is there a Kurchan nxn square such that it is at the same time a magical nxn square, if the square is filled with the first consecutive n^2 a) integers, b) primes, c) X-type numbers?

Q5. Is there a n value such that K°(n)=0, if the square is filled with the first consecutive n^2 a) integers, b) primes, c) X-type numbers?

Solution:

Contributions came from **Luke Pebody, J. K. Andersen and Carlos Rivera.**

Luke Pebody confirmed that the K°(4) obtained by **C. Rivera** is correct (the best possible).

Carlos Rivera improved his own solution for K(5) from 3680 to 2610 (can you find what is this improved arrangement?)

J. K. Andersen wrote, for the question 5:

No for a) and b). Yes for c) with certain X-types, e.g. powers of an arbitrary number.

> By definition, K0(n)=0 means there is a square where all rows, columns and diagonals have the same product. Two products are the same if and only if they have the same prime factorization, so every prime must appear the same number of times in the factorization of each row, column and diagonal. This means the multiplicity (exponent) of each prime must form a magic (sum) square. It does not have to be the same magic square for different primes.
>
> Example: Show that K0(3)=0 for X-type = powers of 2.
>
> Start with an arbitrary magic square with numbers from 0 to 3^2-1, e.g.:
>
> 7 0 5
> 2 4 6
> 3 8 1
>
> Raise an arbitrary integer to these powers, e.g. 2:
>
> 2^7 2^0 2^5
> 2^2 2^4 2^6
> 2^3 2^8 2^1
>
> And get a square with constant product 2^12=4096:
>
> 128 1 32
> 4 16 64
> 8 256 2
>
> In general: X-type has K0(n)=0 if and only if the first n^2 X-type numbers have prime factorizations where the multiplicity for each prime factor can form a magic n*n square.

***

Carlos Rivera improved (2/2/04) his own solution for K(5) from 2610 to 2052 (can you find what is this improved arrangement?)

***

Anurag Sahay wrote (May, 2005):

For Q3, I found a solution better than your third best
: k(5) = 3474

10 20 22 25 1
5 4 12 24 19
8 14 23 2 21
15 11 6 7 16
18 9 3 13 17

***

Anurag Sahay wrote (Set. 05):

Q3 of puzzle 252: I improved the value of k(5) to 3168.

10	23	24	1	20
2	19	17	21	8
18	12	4	25	5
14	3	11	16	15
22	7	6	13	9

***

On Set 26, 05, Luke Pebody reported:

k(5)=1744, This is the best solution... I have searched all ranges [m, m+1, ..., n] where m,n are products of five numbers in the range [1-25], n-m<1744 and m^5<25!, n^5>25! for possible squares, and there is no such range.

The square will be published later on Anurag's request.

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Problems & Puzzles: Conjectures

Conjecture 79. **Rodolfo Kurchan's Conjecture**

Claudio Meller, in his always intersting site, posted the entry 1472 related to the following conjecture original from Rodolfo Kurchan:

> "The most of the integers may be expressed as a sum of two palindromes. These few integers that do not fit the previous rule, may be expressed as a sum of one normal palindrome and another 'special palindrome' that accepts k zeros to the left of a central normal palindrome and ends in k zeros to the right of the central palindrome" **(Original Version)**

Examples:

> a) Using two normal palindromes:
> 2017 = 1331 + 686
> 20149580973 = 19869096891 + 280484082
>
> b) Using a normal palindrome and a special palindrome:
> 2001 = 1001 + 0001000
> 20201 = 11111 + 09090
> 2073 = 363 + 01710
> 91729 = 91619 + 0110

Claudio and Rodolfo asked for counterexamples:

Carlos Rivera found the earliest counterexample: 1200 can not be expressed as the conjecture is expressed, but needs two special palindromes: 1200 = 00100 + 001100.

After this, Claudio and Rodolfo asked if the new conjecture:

> "Any integer may be expressed a sum of two palindromes, both normal or one normal and another special or two special palindromes" **(Second version)**

had a counterexample. This time a puzzler named "Mmonchi" found the earliest counterexample: 113001. He also sent 100 other counterexamples after 113001.

At this point Carlos Rivera observed that perhaps the 2nd Rodolfo's conjecture might be saved this way:

> "Any integer may be expressed as an algebraic sum of two palindromes, both normal or one normal and another special or two special palindromes" **(Third version)**

because himself obtained the following solution 113001 = 0204020 - 91019

I have been told by Claudio Meller that Mmonchi has tested all the integers less than 10^5 and has verified that all of them satisfy this third version, but he can not test neither larger integers nor obtain a positive proof of the general validity of this 3rd version of the Rodolfo's conjecture

Q. Can you obtain a proof or a counterexample of the 3rd version of the Rodolfo's Conjecture?

Puzzle 259. Not dividing any pandigital

Rodolfo Kurchan recently asked to me the following question:

> ***"What is the smallest number not dividing any 10 digits-pandigital?"***

After I solved his question I asked to him in return:

> ***"What is the smallest prime number not dividing any 10 digits-pandigital?"***

Q1. Can you solve both questions without considering an exhaustive test of all the pandigital numbers?

Q2. Redo the exercise with the 9-digits pandigital (zero-free) numbers?

______

* *10 digits-pandigital* is a number of 10 digits having all the decimal digits from 0 to 9.

Solution:

Faride Firoozbakht and Patrick de Geest sent contributions to this puzzle.

Faride solved the **Kurchan's** original question for pandigital numbers the 10 digits and for zero-free pandigital numbers of 9 digits: 100 and 10, respectively.

Faride and **Patrick de Geest** found the prime-solutions to the **Rivera's** question but not satisfying the condition of the puzzle (not testing ALL the pandigitals in any case). So I will not show them yet.

***

Puzzle 263. MagicAntiMagic Squares

The past Monday, (April 5, 2004) **Rodolfo Kurchan** sent to me by email the following nice 5x5 **antimagic square**:

					59
7	**8**	**24**	**22**	**2**	63
4	**16**	**9**	**14**	**21**	64
25	**11**	**13**	**15**	**5**	69
6	**12**	**17**	**10**	**23**	68
18	**20**	**3**	**1**	**19**	61
60	67	66	62	70	65

The stunning feature of this antimagic 5x5 square is that contains an embedded centered nut of a 3x3 magic square (numbers in **red** color); that is to say here we have **a magic square inside an antimagic square!** A beauty example of an object that contains within its contrary.

Perhaps the beauty of kind of this objects is excuse enough in order to keep away the prime numbers for this puzzle.

Questions:

1. Can you get another antimagic solution of the same size (5x5) using a distinct magic (3x3) square?

2. Can you get an antimagic 5x5 containing an eccentric (non-centered) magic 3x3

3. Can you get a larger example (i.e. an antimagic 6x6 containing a magic 4x4)?

4. Can you get the opposite concept: a magic square containing inside an antimagic square (*)

(*) It has been shown that no antimagic square of order less than 4 can exist; then, the minimal example of this kind of objects could be an magic square 6x6 containing a centered antimagic square 4x4.

Solution:

I feel my self really happy because the readers of my pages are more daring than I suppose. While I was thinking that this puzzle was hard enough in order to add the primality condition of the numbers used, **J.C. Rosa** got a solutions to Question 4 using only primes! Here is what he wrote:

It is possible to find one 3x3 antimagic square with prime numbers (see Won plate 132) and particularly for your puzzle 263 I have found this :

The following antimagic square is composed of nine primes with its eight sums in arithmetic progression (step 2). The sums go from 443 up to 457:

101	113	233
293	151	13
59	191	199

(note that the central number is a palprime)

And now the same antimagic square embedded in a 5x5 magic square:

29	157	277	263	23
229	101	113	233	73
11	293	151	13	281
197	59	191	199	103
283	139	17	41	269

magic sum = 749

(note that this puzzle is the puzzle 263 and 263 is inside this square)

***

J. C. Rosa added:

do you want an 3x3 prime antimagic square embedded in a 5x5 prime magic square with a prime magic sum ?

Here it is ::

83	43	139	23	101
151	29	113	89	7
41	149	79	13	107
53	59	47	127	103
61	109	11	137	71

For the 3x3 antimagic square the sums go from 227 up to 241 (step 2). For the 5x5 magic square the magic sum is 389 (prime)

***

J. C. Rosa also contributes to Q1:

About the question 1 of the puzzle 263 there are a lot of different solutions . Here are two examples with the numbers from 1 to 25 and one example with only 25 prime numbers (unfortunately they are not consecutive).See below. Now I hope to find a bridge between the question 1 of the puzzle 263 and the question 2 of the puzzle 264 : A 3x3 magic square embedded in an 5x5 antimagic square composed only 25 consecutive primes....

5	7	20	18	10
17	2	23	14	6
4	25	13	1	21
19	12	3	24	11
16	22	8	9	15

Magic sum=39 . The sums go from 59 to 70

xxxxxxxxx

5	14	18	17	8
15	2	21	13	10
4	23	12	1	25
24	11	3	22	7
20	16	9	6	19

Magic sum=36 . The sums go from 59 to 70

xxxxxxxxxxxxx

67	31	23	107	61
7	17	89	71	109
41	113	59	5	83
127	47	29	101	3
53	97	103	13	43

Magic sum=177 The sums go from 287 to 309 (step 2)

***

J. C. Rosa contribution to Q2 arrived the 12/6/04:

About the question 2 of the puzzle 263 I have found many solutions. Here are only three examples (the magic squares are in bold letters at the top left corner)

a) with the numbers from 1 up to 25 :

2	21	13	19	5
23	12	1	18	7
11	3	22	10	20
15	4	25	9	17
16	24	8	6	14

Magic sum=36 . The sums of the antimagic 5x5 go from 59 up to 70.

b) with 25 prime numbers (they are not consecutive) :

17	89	71	109	43
113	59	5	53	103
47	29	101	7	157
37	3	151	137	19
131	163	11	31	13

Magic sum=117 . The sums of the antimagic 5x5 go from 327 up to 349 (step 2).

c) (the best till the end !) with 25 CONSECUTIVE PRIME NUMBERS :

(moreover this example is a solution of the question 2 of the puzzle 264)

41	89	83	79	37
113	71	29	73	61
59	53	101	31	97
109	103	47	23	43
17	19	67	127	107

Magic sum=213 . The sums of the antimagic 5x5 go from 325 up to 347 (step 2).

***

For the question 3, **Rodolfo Kurchan** wrote (Feb 18, 2005):

In 2005 I found an **antimagic** 6x6 square that contains in the center a 4x4 **magic square**:

						108
1	**36**	**34**	**33**	**2**	**3**	**109**

35	**26**	**13**	**12**	**23**	**6**	**115**
27	**15**	**20**	**21**	**18**	**5**	**106**
10	**19**	**16**	**17**	**22**	**30**	**114**
9	**14**	**25**	**24**	**11**	**29**	**112**
31	**7**	**8**	**4**	**28**	**32**	**110**
113	**117**	**116**	**111**	**104**	**105**	**107**

***

Anurag Sahay had previously sent (Jan 2005) the following solutions to Q3:

> Some solutions for Q3:
>
> 5 32 29 3 6 30
> 27 12 19 18 25 8
> 36 26 24 11 13 7
> 9 15 17 22 20 31
> 34 21 14 23 16 2
> 4 10 1 35 33 28
>
> 7 29 34 3 9 28
> 30 12 19 18 25 10
> 6 26 24 11 13 31
> 35 15 17 22 20 8
> 33 21 14 23 16 2
> 5 1 4 36 32 27
>
> 6 7 34 29 32 2
> 33 12 19 18 25 10
> 4 26 24 11 13 36
> 8 15 17 22 20 27
> 30 21 14 23 16 1
> 31 35 5 3 9 28
>
> 4 3 34 36 31 2
> 28 11 25 12 26 9
> 1 22 20 17 15 30
> 8 23 13 24 14 35
> 33 18 16 21 1

***

Puzzle 457. Prime word embedded.

Rodolfo Kurchan sent the following nice puzzle:

> Find the smallest prime number whose name written in English contains the letters "PRIME" in order embedded in it.
>
> **Q1. Solve the Rodolfo question.**
> **Q2. Redo it in your origin language.**

Frederick Schneider wrote:

Q1. For American and Modern British English, the answer is:

1000000000000000035000061: one sePtillion thiRty-fIve Million sixty-onE

In the "Traditional British" and "Traditional European" numbering, the first number containing a P is also sePtillion but sePtillion is defined as 10^42 in these systems. The rest of the letters can be found in a similar way.

The smallest prime is 10^42 + 35000007: one sePtillion thiRty fIve Million sEven.

> Q2. Assuming you don't want to translate PRIME into another language, for Indonesian, the answer is much smaller (perhaps the smallest?): 859 = delaPan-Ratus LIMa-puluh sEmbilan (eight hundred fifty-nine)
>
> (**What if we want to translate "Prime" to Indonesian?**. Frederick responded: The original word for prime is "ganjil" but there's no numbers in Indonesian that contain the letter g, so...)

***

Luke Pebody wrote:

Q1. Is it "one sePtillion thiRty-fIve Million sixty-onE?"

> I am pretty sure that the P does not appear in anything below one septillion, that the M can come from nothing smaller than a million, and that thirty five is the smallest number to contain R and I in that order. This shows a lower bound of one septillion and thirty five

million. The next prime above 10^24+35*10^6 is 10^24+35*10^6+61.

If you speak British, then a septillion is 10^42, so the answer is 10^42+35*10^6+7.

***

Carlos Rivera wrote:

Q2. I found in Spanish "Prime" or "Primo" in the same smallest prime number:
10^42+30*10^3+21 = Septillón Treintamil Veintiuno

***

J. C. Rosa wrote:

About Q2 , in French : "PRIME"="PREMIER" , I found the following result: 704003=sept cent quatre mille trois. I think that this prime number is the smallest in French.

***

Patrick Capelle wrote:

Q2, in French.

Word « PRIME »:
73009 = septante-trois mille neuf *
703013 = sept cent trois mille treize **

Word « PREMIER »:
74047 = septante-quatre mille quarante-sept *
704003 = sept cent quatre mille trois **

* Belgium, Suisse romande, Val d'Aoste, East of France, République démocratique du Congo, Rwanda.
** France

***

Nick McGrath wrote (Aug., 08):

Just for amusement I tried some other embedded names of numbers.
The smallest I could come up with were:

COMPOSITE
10^117 + 10^60 + 10^24 +16
= one oCtOtrigintillion one noveMdecillion one sePtilliOn SIxTEen

SQUARE
6*10^15 + 11*10^6 + 750*10^3 + 889

=Six QUAdRillion Eleven million seven hundred fifty thousand eight hundred eighty nine

NATURAL
oNe quAdrillion Twenty foUR thousAnd eLeven.

How about TRIANGLAR, FACTORIAL, FIBONACCI, EMIRP etc?

***

About the Nick's contribution my comment is this:

Find the smallest X-number such that when written in L-language the word X appears in order. X= Square, Triangular, Fibonacci, etc. (See Puzzle 459)

***

Rodolfo Kurchan wrote (Aug., 08):

Te mando las soluciones que recibí en Snark de **Jaime Rudas** de Bogotá:

> Creo que encontré una solución para el número primo en alemán, con las letras en orden:
>
> HAUPTZAHL (número primo)
>
> HundertAchtUndzwanzig sePTillionen
> achtZigtAusendacHthunderteLf
> 128 x 10^42 + 80811
>
> En Portugués:
>
> PRIMO
>
> um sePtilião e tRInta Mil centO e cinquenta e um
>
> En Ruso:
>
> ПРОСТОЕ
> PROSTOE
> 531 163 - Пятьсот тРидцать Одна тыСяча сТО шЕстьдесят три
> 531 163 - Pyat'sot tRidtsat' Odna tySyacha sTO shEst'decyat' tri
>
> En Holandés:
>
> PRIEMGETAL
> één sePtiljoen dRIE Miljoen honderdneGEnenTAchtigduizend achthonderdeLf
> 10^42 + 3 189 811

***

Gennady Gusev wrote (Set 08)

Hi,
I would like to suppose my solutions of the puzzles 457 and 459 for Russian language.
In Russian there are no letters U (У) and F (Ф) in name of numbers so Natural, Fibonacci, Triangular and Factorial are impossible. The term 'OTHER' is not used.

Solutions:

5399
Пять тысяч тРиста девянОСТО дЕвять
Pyat' tysyach tRista devyanOSTO dEvyat'

COMPOSITE - СОСТАВНОЕ - SOSTAVNOE
40394
СОрок тыСяч ТристА деВяНОсто чЕтыре
SOrok tySyach Trista deVyaNOsto chEtyre

SQUARE - КВАДРАТ - KVADRAT
40373316
сороК миллионоВ тристА семьДесят тРи тысячи тристА шесТнадцать
soroK millionoV tristA sem'Desyat tRi tysyachi tristA shesTnadtsat'

EMIRP - ЕОТСОРП - EOTSORP
740153
сЕмьсОТ СОРок тысяч сто Пятьдесят три
sEm'sOT SORok tysyach sto Pyat'desyat tri

***

Claudio Meller wrote (March 2011):

Solutions in Spanish

Smaller number with the letters of Natural :
1.440.000 uN millón cuATrocientos cUaRentA miL

Smaller number with the letters of Cuadrado (square in spanish) :
4.231.249: CUAtro millones Doscientos tReintA y un mil DOscientos cuarenta y nueve

Smaller number with the letters of Square :
2514: doS mil QUinientos cAtoRcE

Smaller number with the letters of Emirp :

trEs MIl tRes sePtillones

Smaller number with the letters of Composite :
CuatrO Mil sePtillones Ochenta y SIeTE

Smaller number with the letters of Compuesto (composite in spanish):
CuatrO Mil sePtillones cUatrociEntoS cuaTro

***

Coll.20th-015. Consecutive primes and Pandigitals

On May 6, 2018, Rodolofo Kurchan, wrote:

Q. Find k consecutive primes that when added produces

a) the smallest ten digits pandigital.

b) the largest ten digits pandigital.

Do this for k=2, 3, 4,..., 10.

Send a solution

Contributions came from Jeff Heleen, Claudio Meller and Emmanuel Vantieghem

***

Jeff wrote on Set 3, 2018:

For Coll.20th-015 I have
Q1:
k = 2: 511729877 + 511729891 = 1023459768
k = 3: 341152541 + 341152571 + 341152577 = 1023457689
k = 4: 255864403 + 255864407 + 255864437 + 255864451 = 1023457698
k = 5: 204697291 + 204697303 + 204697327 + 204697333 + 204697343 = 1023486597
k = 6: 170579861 + 170579897 + 170579939 + 170579951 + 170579957 + 170579963 = 1023479568
k = 7: 146208341 + 146208353 + 146208367 + 146208371 + 146208407 + 146208421 + 146208437 = 1023458697
k = 8: 127933427 + 127933433 + 127933453 + 127933499 + 127933501 + 127933513 + 127933549 + 127933583 = 1023467958
k = 9: 113738491 + 113738507 + 113738531 + 113738621 + 113738641 + 113738671 +

113738683 + 113738701 + 113738743 = 1023647589
k = 10: 102357823 + 102357847 + 102357859 + 102357863 + 102357881 + 102357887 + 102357919 + 102357953 + 102357961 + 102357971 = 1023578964
Q2:
k = 2: 4938271579 + 4938271631 = 9876543210
k = 3: 3292174651 + 3292174687 + 3292174693 = 9876524031
k = 4: 2469112747 + 2469112817 + 2469112829 + 2469112837 = 9876451230
k = 5: 1975300447 + 1975300451 + 1975300493 + 1975300511 + 1975300529 = 9876502431
k = 6: 1646088649 + 1646088673 + 1646088677 + 1646088683 + 1646088707 + 1646088751 = 9876532140
k = 7: 1410934643 + 1410934667 + 1410934687 + 1410934717 + 1410934739 + 1410934781 + 1410934787 = 9876543021
k = 8: 1234542527 + 1234542553 + 1234542607 + 1234542623 + 1234542637 + 1234542679 + 1234542707 + 1234542719 = 9876341052
k = 9: 1097393513 + 1097393519 + 1097393527 + 1097393543 + 1097393551 + 1097393581 + 1097393617 + 1097393623 + 1097393629 = 9876542103
k = 10: 987650239 + 987650263 + 987650311 + 987650317 + 987650341 + 987650369 + 987650371 + 987650387 + 987650401 + 987650413 = 9876503412

***

Claudio wrote on Set 3, 2018:

Smallest

	First Prime	Pandigital
2	511729877	1023459768
3	341152541	1023457689
4	255864403	1023457698
5	204697291	1023486597
6	170579861	1023479568
7	146208341	1023458697
8	127933427	1023467958
9	113738491	1023647589
10	102357823	1023578964

Largest

	First Prime	Pandigital
2	4938271519	9876543012
3	3292174651	9876524031
4	2469112837	9876451230
5	1975300529	9876502431
6	1646088649	9876532140
7	1410929137	9876504231
8	1234542527	9876341052
9	1097393513	9876542103
10	987650239	9876503412

***

Emmanuel wrote on 7-9-18:

It was easy to find solutions for k = 2 to 10 :

Lowest :

k	first prime	first pandigital
2	511729877	1023459768
3	341152541	1023457689
4	255864403	1023457698
5	204697291	1023486597
6	170579861	1023479568
7	146208341	1023458697
8	127933427	1023467958
9	113738491	1023647589
10	102357823	1023578964

Highest

k	first prime	last pandigital
2	4938271579	9876543210
3	3292174651	9876524031
4	2469112747	9876451230
5	1975300447	9876502431
6	1646088649	9876532140
7	1410934643	9876543021
8	1234542527	9876341052
9	1097393513	9876542103
10	987650239	9876503412

It was even relatively easy to do this for all k <= 26365.
For k = 26366 there is no solution. I. e. : no sum of 26366 consecutive primes is (a 10-digit) pandigital.
For k = 26365 :

Lowest : First prime : 353. Sum : 3842075961
Highest : First prime : 143821. Sum : 8104596723

***

Problems & Puzzles: Problems

Problem 66. Every positive integer is the sum of 3 palindromes. Looking for another proof.

"La elegancia en matemáticas no es indispensable, pero se agradece". Ramón David Aznar.

On February 2016, **Javier Cilleruelo and Florian Luca** published a demonstration of the following theorem:

> **Let g ≥ 5. Then any positive integer can be written as a sum of three base g palindromes.**

The proof, according to the experts, is correct. The proof is "*algorithmic*".

Perhaps the two best values of the proof are:

> a) It improves the W. D. Banks previous result (2015), who demonstrated that "*every positive integer can be written as a sum of at most 49 base 10 palindromes*".
>
> b) Being algorithmic, it provides a mean to compute at least one solution.

In another place, Cilleruelo wrote "*El algoritmo que utilizamos es complejo pero elemental, en el sentido que no se utilizan matemáticas profundas... la casuística es tan compleja que hace que el artículo se alargue hasta las 39 páginas*"

Yes, indeed.

For me, the proof from Cilleruelo & Luca is still not awful but ugly. Why?

In short, the demonstration divides the integers in "**small** integers" (6 or less digits) and "**large** integers" (7 or more digits). The large integers are divided in two types "**normal** large integers" and "**special** large integers". For the solution of the "normal large integers" there are 4 algorithms. For the solution of the "special large integers" there is a 5th algorithm. For the small integers there are more that 22 schemes of solution...

Q1. Is someone out there that could attempt to simplify the proof, that is to say, to reduce the quantity of algorithms and schemes to get one solution for every integer?

Q2. What if we change to the following statement: "Any positive integer can be written as an algebraic sum of three palindromes, base 10"? Is this statement easier to probe and compute than the original one from Cilleruelo & Luca?

Here we are trying to follow the lucky fate of the Kurchan's conjecture, but without using the "special palindromes" used there. See Conjecture 79.

Emmanuel Vantieghem wrote on March 03, 2017:

I cannot answer Q1 and just give a partial answer to Q2.

But, using Dmitri's proof of the theorem that every number is the difference of two special palindromes, I can prove in a simple way :

"Every number is the algebraic sum of four (normal) palindromes."

Indeed,
Let s be a special palindrome. Say, s = an n-digit palindrome p followed by k zeros or s = $p(0)_k$ Then it is easily seen that s is the difference of two normal palindromes: s =$p(0)_k= (1)_k p(1)_k - (1)_k(0)_n(1)_k$, where n is the number of digits in p.

(example : 364546300 = 11364546311 - 11000000011).

Now, any number m can be written as s1 - s2, two special palindromes.
Since s1 = a1 - b1 (two palindromes) and s2 = a2 - b2 (also two palindromes), we have m = a1 - b1 - a2 + b2, QED.

Of course, if m is not divisible by 10, then Dmitri's theorem states that m is the difference of a normal palindrome and a special palindrome. In this special case m can be written as an algebraic sum of **three** normal palindromes.

***

Carlos Rivera applies the Dmitry's algorithm and the Emmanuel's ideas to the following two examples. Both examples come from the Cilleruelo and Luca paper:

Example #1 (p.12), m@10<>0

m=314159265358979323846 (21 digits) =
+6092587554049587359044409537859404557852906 (43 digits, NP)
-6092587554049587359044095378594045578529060 (43 digits, SP)
=
+6092587554049587359044409537859404557852906 (43 digits, NP)

-(16092587554049587359044095378594045578529061-
10000000000000000000000000000000000000000001)

Example #2 (p.14), m@10=0

m=2718281828459045235360 (22 digits) =
+6948060106091020313033303130201906010608496 0 (44 digits, SP)
-694806010609102031303330313020190601060849600 (44 digits, SP)
=
+(16948060106091020313033303130201906010608496 1- 1000000000000000000000
0000000000000000000001)
-(1169480601060910203130333031302019060106084961 1-
110000000000000000000000000000000000000000011)

NP stands for Normal Palindrome; SP stands for Special Palindrome.

Puzzle 267. Talisman Squares

"The study of these squares is so new, in fact, that no rules for construction are known, nor are there any mathematical theories...". **Joseph S. Madachy**, 'Madachy's Mathematical Recreations', 1966.

A square of order n, filled with the integers from 1 to n^2, has a '**Talisman constant**' equal to the minimal difference between each of its elements and the immediate neighbors (diagonal ones included) to each one of it.

But not all the squares are '**Talisman Squares**'. This name is deserved for those squares that, for a given order n, has a maximal Talisman constant.

Example. For n=4

16	3	2	13
5	10	11	8
9	6	7	12
4	15	14	1

9	5	11	7
13	1	15	3
10	6	12	8
14	2	16	4

The left one is the well known magic square named "Dürer Magic Square" and has a Talisman constant equal to 1, while the right square has a Talisman constant equal to 3.

So the left one square can not be a Talisman Square for this order (4), while we may assert (proof?) that the right one square certainly is a Talisman Square, because 3 is the largest Talisman constant possible for any square of order 4.

But, **how to construct a Talisman Square for any given order n**?

Rodolfo Kurchan and **Carlos Rivera** have been studying this problem the last two months, and they have found a pair of algorithms (one algorithm for even n values, the other one for odd n values) in order to produce (**conjecturally**) the asked Talisman squares for any given order n.

Instead of providing long-winded and boring general instruction rules, we will display the algorithms using a couple of examples and some explanations about them.

n = even, $K_{TS}(n) = n^2/4 - 1$

Example n=6, $K_{TS}(6) = 8$

19	10	22	13	25	16
28	1	31	4	34	7
20	11	23	14	26	17
29	2	32	5	35	8
21	12	24	15	27	18
30	3	33	6	36	9

As you have noticed, for sure, the filling pattern exhibited in the previous example, divides the numbers 1, 2, 3, …, n^2 into four sets (S1, S2, S3, S4) of $n^2/4$ consecutive numbers each, as follows:

S1 = {1, 2, 3, …, X-1}
S2 = {X, X+1, X+2, …, Y-1}
S3 = {Y, Y+1, Y+2, …, Z-1}
S2 = {Z, Z+1, Z+2, …, n^2}

The $n^2/4$ consecutive numbers of each set are allocated in the same general trend:

> Starting from certain specific position inside the four cells of the upper-left corner, the rest of the numbers of each set are allocated consecutively '*every two cells downward and rightward*'. In the same moment you finish allocating the last number of the first set you know what is the first number of the following set, and so on.

So, the only important thing you should know in advance is the cells in which the first numbers of each set (1, X, Y & Z) must be allocated, and the answer is: 1 goes in the cell (2, 2), X goes in the cell (1, 2), Y goes in the cell (1, 1) and Z goes in the cell (2, 1). Moreover, if you know to know them in advance, the values of X, Y and Z are, respectively $n^2/4+1$, $2.n^2/4+1$ and $3.n^2/4+1$, but this is not necessary at all.

Y	X
Z	1

We will call this filling pattern "**22A**" (22 because it starts in the cell (2, 2) and "A" because the four starting numbers of each set - 1, X, Y & Z - describes the profile of an "A" letter)

n = odd, K_{TS}(n) = (n.(n-1))\4 (*)

Example: n=7, $K_{TS}(7) = 10$

13	40	17	32	21	36	25
1	29	4	44	7	47	10
14	41	18	33	22	37	26
2	30	5	45	8	48	11
15	42	19	34	23	38	27
3	31	6	46	9	49	12
16	43	20	35	24	39	28

As before, here are four sets (S1, S2, S3 & S4) of consecutive numbers. Now the four sets have distinct quantity of integers. Again, the starting number of each set, 1, X, Y & Z are allocated in the four cells in the upper-left corner, but now 1 goes in the cell (2, 1), X goes in the cell (1, 1), Y goes in the cell (2, 2) and Z goes in the cell (1, 2). We will call this pattern "**21N**" for analogue reasons than before.

X	Z

1	Y

The consecutive numbers of the four sets, are allocated in the same general trend than before: '*every two cells, downward and rightward*'.

But we have a very important difference:

> When you allocate the numbers of the set S3, since the column 4+2.(c\4-1) you will shift upward one cell all the cells that will receive the corresponding numbers for this column. The same will happen with all the columns rightward of this column.
>
> Consequently, when you allocate the numbers of the set S4, since the column 4+2.(c\4-1) you will shift downward one cell all the cells that will receive the corresponding numbers for this column. The same will happen with all the columns rightward of this column.

Summarizing:

Talisman squares are constructed this way:

For n even:

> Use the filling pattern **22A** (**) for the starting numbers (1, X, Y & Z) of the four sets (S1, S2, S3 & S4) of n2/4 consecutive numbers; allocate each integer of every set, using the general procedure '*every two cells downward, rightward*'. Proceeding this way, $K_{TS}(n) = n^2/4 - 1$.

For n odd:

> Use the filling pattern **21N** for the starting numbers (1, X, Y & Z) of the four sets (S1, S2, S3 & S4) of consecutive numbers; allocate each integer of every set, using the general procedure '*every two cells downward, rightward*'. For the sets S3 & S4 you will need to shift upward and downward, respectively, the starting cell in each column equal or grater than 4+2.(c\4-1). Proceeding this way,
>
> $K_{TS}(n) = (n \cdot (n-1))\backslash 4$

n, order of a Talisman Square.	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**
$\mathbf{K_{TS}(n) = n^2/4 - 1}$, for n=even;	1	3	5	8	10	15	18	24	27

K_{TS}**(n) = (n·(n-1))\4** for n= odd									
First shifted column, **4+2.(c\4 -1)**, sets S3 & S4, just for n odd	-	-	4	-	4	-	6	-	6

Question:

Can you produce a square of any order with a talisman constant greater than the predicted by our algorithms?

(*) " \ " is the symbol for integer division.

(**) As a matter of fact, for the squares of order n even, we have found two more general patterns that produce the same Talisman constant. We have selected this pattern (22A) because it seems appropriate in order to produce Talisman rectangles also. Nevertheless this is a work still in process.

Solution:

Contribution came from **Luke Pebody**. On May 18 he wrote " I proved that no talisman square can be produced for even n, with Talisman Constant at least (n^2/4)-1, so that you got the correct answer". This his proof:

> Split the nxn grid into (n^2/4) 2x2 grids. The difference between any pair of squares in any of these subgrids is at most t. The smallest element of one of the grids is at least (n^2/4). Therefore (n^2/4)+3t<=n^2. Therefore t<=n^2/4.
>
> If t=n^2/4, this argument shows that each of these 2x2 grids contains numbers {k,k+t,k+2t,k+3t} for some 1<=k<=t.
>
> Now, let the square for {k,k+t,k+2t,k+3t} and {l,l+t,l+2t,l+3t} with k<l be orthogonally adjacent. Then there are a pair of squares from {k,k+t,k+2t,k+3t} adjacent to a pair from {l,l+t,l+2t,l+3t}. If k+it is adjacent to l+jt, then i!=j and i!=j+1. Therefore the adjacency must be k,k+t are adjacent to l+2t,l+3t. Therefore, looking at the square corresponding to k=1, 1 and 1+t must be on each internal edge of that square.
>
> ...

Let n=20, just to express the point clearly.

We have split the 20x20 grid into 100 2x2 subgrids. Each of them has a different smallest element. Therefore 1 of them must have smallest element at least 100. Therefore, if k is the talisman constant of the whole thing, that small grid must contain numbers of size at least 100,100+k,100+2k and

100+3k. Thus 100+3k<=400.

(And what about the odd case?) No idea, I'm afraid. Too difficult.

Puzzle 1027. Integers as sum of distinct repdigits

Rodolfo Kurchan sent the following nice puzzle

We have up to nine repdigits numbers, from 1 to 9.
Each number can have 0<=N<10 equal digits. It is invalid to add two integers with the same digit even if these two have distinct quantity of digits. Example 8888 y 888 can not be used in the same expression.

Example with a solution:

98765 = 88888 + 7777 + 1111 + 555 + 333 + 99 + 2

On the other hand, I think that 987654 o 987650 are impossible to be expressed as said above.

Q1. What is the minimal integer impossible to be expresses as said.

Q2. Redo Q1 for prime numbers.

Q3. Redo Q1 for zero-free pandigitals

In any case show your solutions for the three numbers of each type previous to the one without solution

From Jan 9-15, 2021, contributions came from Emmanuel Vantieghem, Paul Cleary, Oscar Volpatti

***

Emmanuel wrote:

Q1.

The smallest number not representable as a sum of repdigits is 25427.
Solutions for the three previous numbers are :
25424 = 11111 + 222 + 3333 + 5 + 666 + 88 + 9999
25425 = 11111 + 2222 + 3333 + 444 + 555 + 6666 + 7 + 88 + 999
25426 = 11111 + 2222 + 333 + 4444 + 555 + 6666 + 7 + 88

Q2.
The smallest prime not representable as a sum of repdigits is 32027.
Solutions for the three previous prime are :
31991 = 11111 + 2222 + 3333 + 5555 + 6 + 777 + 8888 + 99
32003 = 11111 + 2222 + 3333 + 4444 + 6 + 888 + 9999
32009 = 11111 + 2222 + 3333 + 4444 + 5 + 7 + 888 + 9999

Q3.
The smallest zero-free pandigital is 123456789 and it has no representation as a sum of repdigits.

(Not asked : the smallest representable zero-free pandigital is :
123457896 = 11111 + 2222 + 33 + 444444 + 55555555 + 66666666 + 777777 + 88
and the biggest :
984673251 = 11111111 + 222222 + 33 + 5555 + 6666666 + 77777777 + 888888888 + 999)

***

Paul wrote:

Q1.

The minimum number is 25427.

25424 = 2 + 33 + 4 + 55 + 6666 + 7777 + 888 + 9999
25425 = 1 + 2 + 33 + 4 + 55 + 6666 + 7777 + 888 + 9999
25426 = 1111 + 22222 + 444 + 555 + 7 + 88 + 999

Q2.

The minimum prime is 32027.

31991 = 22222 + 5 + 777 + 8888 + 99
32003 = 22222 + 444 + 555 + 6 + 7777 + 999
32009 = 22222 + 3 + 44 + 66 + 777 + 8888 + 9

Q3.

The minimal number is already the smallest, so here are the first 3 pan digitals that can be made with repdigits.

123457896 = 11111 + 2222 + 33 + 444444 + 55555555 + 66666666 + 777777 + 88
123458967 = 11111 + 222222 + 3333 + 4 + 55555555 + 66666666 + 77 + 999999
123458976 = 11111 + 2 + 3333 + 444444 + 55555555 + 66666666 + 777777 + 88

***

Oscar wrote:

Below 111111, there are 149 numbers which can't be expressed as required; 19 of them are primes.

About Q1.
25424 = 2+33+4+55+6666+7777+888+9999
25425 = 111+22+3+4+5555+66+777+8888+9999
25426 = 11111+22+3333+66+7+888+9999
25427 -> no solution.

About Q2.
31991 = 1+22222+33+5+66+7777+888+999
32003 = 1+333+4444+555+6+7777+8888+9999
32009 = 11+2+333+4444+555+7777+8888+9999
32027 -> no solution.

About Q3.
Below 10^9, there are 9! = 362880 zero-free pandigitals, but only 14825 of them can be expressed as required.
In particular, there's no way to express the first nine of them:
123456789, 123456798, 123456879, 123456897, 123456978, 123456987, 123457689, 123457698, 123457869.
These are the first three zero-free pandigitals for which there are solutions:
123457896 = 111111111+2222+33+44444+5555555+6666666+77777+88
123458967 = 111111111+22222+3333+4+5555555+6666666+77+99999
123458976 = 111111111+2+3333+44444+5555555+6666666+77777+88

Problems & Puzzles: Puzzles

Puzzle 1117 2023

At first sight 2023 is a notorious inconspicuous integer, but...

A. On my request **Rodolfo Kurchan** sent the following puzzle about it...

2023 is a composite integer such that abs(20-23)=3 and 20+23=43, both primes!

Under the same scheme the first prime integer is 41 [abs(4-1)=3, 4+1=5]

Q1. Find the smallest pandigital prime as with the same scheme of 2023, having a) the following decimal digits, 1 to 9 and b) having the 0 to 9, decimal digits.

B. From the Giovanni Resta's page we know that:

"***The prime factors of 2023, concatenated, give a palindrome: 71717.***"

Q2. What is the next year that produces a) another palindrome b) a palprime?

Q3. Can you provide another interesting prime property and puzzle related to 2023?

Send a solution

During the week from Jan 1-7, 2023, contributions came from Emmanuel Vantieghem, J-M Rebert, Jim Howell, Oscar Volpatti, Gennady Gusev

***

Emmanuel wrote:

Q1
Here I assumed that pandigitallity means "having at least all digits 1, ..., 9 or 0, ..., 9"
a) p = 1125647893
    11256 + 47893 = 59149, prime
    Abs[11256 - 47893] = 36637, prime
b) p = 100126478593
    100126 + 478593 = 578719, prime
    Abs[100126 - 478593] = 378467, prime.
    (here I assumed the number of digits of p must be even)

Q2. Next palindrome year : 2048 = 2*2*2*2*2*2*2*2*2*2*2
      Next palprime year : 3039 = 3*1013

Q3. a) Find the smallest prime whose sum of digits is 2023 (easy)

b) Find the smallest prime whose square has sum of digits 2023 (hard).

____

Sincere greetings and a nice (9 + 8)-(6*5*4 - 3 +2*1) !

***

Jean-Marc wrote:

Q1.a.The smallest pandigital prime, I found is 1125647893.

1125647893, is a prime such that abs(11256 - 47893) = 36637 and 11256 + 47893 = 59149, both primes !

Q1.b.The smallest pandigital prime, I found is 100126478593.

100126478593, is a prime such that abs(100126 - 478593) = 378467 and 100126 + 478593 = 578719, both primes !

Q2.a.The next year that produces another palindrome (22222222222) is 2048 = 2^11.

Q2.b.The next year that produces another palprime (31013) is 3039 = 3*1013.

Q3. Units-digit 7 occurs in the following results :

From 2023 there are 77 = 7*11 years to 2100.

2 + 0 + 2 + 3 = 7 is prime.

precprime(2023) = 2017

nextprime(2023) = 2027

2027 - 2017 = 17-7

and each prime factor of 2023 is ending with 7. (2023 = 7*17^2)

***

Jim wrote:

For Question 2:

The next years where factorization is a palindrome are:

2048=2*2*2*2*2*2*2*2*2*2*2
2097=3*3*233
2187=3*3*3*3*3*3*3
2319=3*773
2321=11*211
2359=7*337
2401=7*7*7*7
2649=3*883
2701=37*73
3125=5*5*5*5*5
3421 = 11*311 and 11311 is prime

So we need to wait almost 1400 years for the first palprime.

Another palprime (probably not the second smallest) is:

10301 = 10301 = prime

Question 3.

I am not sure if this qualifies as “interesting”, but if we concatenate the puzzle number with 2023, we get 11172023 which is a prime number.

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Oscar wrote:

Q1
Both 2023 and 41 have an even number of digits, which are split into two blocks of equal length.
Under this "equal-length" constraint, the smallest pandigital prime solution is:
1125647893 for the "zeroless" case a;
100126478593 for the "full" case b.

abs(11256-47893) = 36637, prime;
(11256+47893) = 59149, prime;
abs(100126-478593) = 378467, prime;
(100126+478593) = 578719, prime.

Without the "equal-length" restriction, there are smaller solutions; in some cases, digits can be split in k different ways.
"Zeroless" pandigital prime case a.
k = 1, p = 1123754869;
abs(112-3754869) = 3754757, prime;
(112+3754869) = 3754981, prime;
k = 2, p = 1128657349;
abs(112-8657349) = 8657237, prime;
(112+8657349) = 8657461, prime;
abs(1128-657349) = 656221, prime;
(1128+657349) = 658477, prime;
k = 3, p = 1325487629;
abs(132-5487629) = 5487497, prime;
(132+5487629) = 5487761, prime;
abs(132548-7629) = 124919, prime;
(132548+7629) = 140177, prime;
abs(132548762-9) = 132548753, prime;
(132548762+9) = 132548771, prime.

"Full" pandigital prime case b.
k = 1, p = 10123548679;
abs(1012-3548679) = 3547667, prime;
(1012+3548679) = 3549691, prime;
k = 2, p = 10125367849;
abs(10-125367849) = 125367839, prime;
(10+125367849) = 125367859, prime;
abs(101253678-49) = 101253629, prime;
(101253678+49) = 101253727, prime;
k = 3, p = 10274863549;
abs(10-274863549) = 274863539, prime;
(10+274863549) = 274863559, prime;
abs(102-74863549) = 74863447, prime;

(102+74863549) = 74863651, prime;
abs(102748-63549) = 39199, prime;
(102748+63549) = 166297, prime;
k = 4, p = 15272406389;
abs(152-72406389) = 72406237, prime;
(152+72406389) = 72406541, prime;
abs(15272-406389) = 391117, prime;
(15272+406389) = 421661, prime;
abs(1527240-6389) = 1520851, prime;
(1527240+6389) = 1533629, prime;
abs(1527240638-9) = 1527240629, prime;
(1527240638+9) = 1527240647, prime;.

Q2
The prime factors of 2023 are concatenated with repetition and in ascending order, obtaining a composite palindrome:
2023 = 7*17^2,
7.17.17 = 71717 = 29*2473.
Under this "ascending-order" constraint, next few years producing a composite palindrome (case a) are:
2048 = 2^11,
2.2.2.2.2.2.2.2.2.2.2 = 22222222222, repdigit;
2097 = 3^2*233,
3.3.233 = 33233 = 167*199;
2187 = 3^7,
3.3.3.3.3.3.3 = 3333333, repdigit;
Next few years producing a palprime (case b) are more sparse:
3039 = 3*1013,
3.1013 = 31013;
3421 = 11*311,
11.311 = 11311;
4303 = 13*331,

13.331 = 13331.

Without the "ascending-order" constraint, there are smaller solutions.
Case a: 2025 = 3^4*5^2,
3.3.5.5.3.3 = 335533 = 11^2*47*59,
3.5.3.3.5.3 = 353353 = 7*11*13*353,
5.3.3.3.3.5 = 533335 = 5*11*9697.
Case b: 2205 = 3^2*5*7^2,
3.7.5.7.3 = 37573, palprime.

7.3.5.3.7 = 73537 = 151*487, composite.

Q3
The number 2023 can be represented as the difference of two 5-smooth integers:
2023 = 3^4*5^2 - 2,
2023 = 2^11 - 5^2.
Can you find one more 5-smooth representation of 2023, or some proof that no more representations exist?
What about 7-smooth representations of 2023?

A p-smooth number is an integer whose prime factors are all less than or equal to p.

For example
2025 = 3^4*5^2 is 5-smooth,
2048 = 2^11 is 2-smooth, so it is 5-smooth too,
2023 = 7*17^2 is not 5-smooth but it is 17-smooth

***

Gennady wrote:

Q1.
In order for the number to be divided into 2 parts equal in number of digits,
then the number of digits in the number must be even.

a) if both parts have an equal number of digits:
1125647893: abs(11256-47893)=36637 & 11256+47893=59149 - all 3 numbers are primes;
if not:
1123754869: abs(112-3754869)=3754757 & 112+3754869=3754981;

b) if both parts have an equal number of digits:
100126478593: abs(100126-478593)=378467 & 100126+478593=578719;
if not:
10123548679: abs(1012-3548679)=3547667 & 1012+3548679=3549691;

Q2.
a) 2048, 22222222222

b) 3039, 31013

Q3.
Express consecutive primes using only the digits 2, 0, 2, 3 and exactly in this order using the operation signs: +, -, *, /, ^, ! (factorial and multifactorial) and concatenation of digits before the first pass.
Beginning (for example):
2 = 2 + 0 * 2 * 3
3 = -20 + 23
5 = 2 * 0 + 2 + 3
7 = 2 + 0 + 2 + 3
11 = 2 + 0! + 2^3 and so on.